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MATHEMATICS OF COILING IN CYLINDRICAL ELECTROCHEMICAL CELLS: THE THEORY OF A SPIRAL BOUNDED BY TWO CIRCLES AND ITS APPLICATION TO THE SPIRAL-WOUND NICKEL–CADMIUM CELL

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Summary

The mathematics relating to a spiral bounded by two circles has been developed and then applied to the spiral-wound nickel–cadmium cell. Given the input parameters, internal can diameter, component thicknesses and mandrel radius the maximum length of each component may be calculated for 16 different variations.

1. Introduction

A coil pack in an electrochemical cell is usually wound on a cylindrical mandrel and fits into a can with circular cross-section. A section normal to the axis of either the mandrel or the can therefore shows each component following the locus of a spiral. In order to maximize cell capacity the gap between the spiral-wound pack and the can should, as far as possible, be filled with active material by overlapping the electrodes.

The relevant mathematics of a spiral bounded by two circles is presented in this paper. This is then applied to the coil pack of the nickel–cadmium cell so that electrode overlap and maximum component lengths may be determined.

2. General mathematics

The equation for a spiral (Fig. 1) as given by Marincic [1] is:

$$\rho_n = r + nt \tag{1}$$

ρ_n is the distance of the spiral from the centre. The spiral originates on the circumference of a circle radius r and moves out a distance t for each turn; n is the number of turns. The length of the spiral (L) is given by [1]:

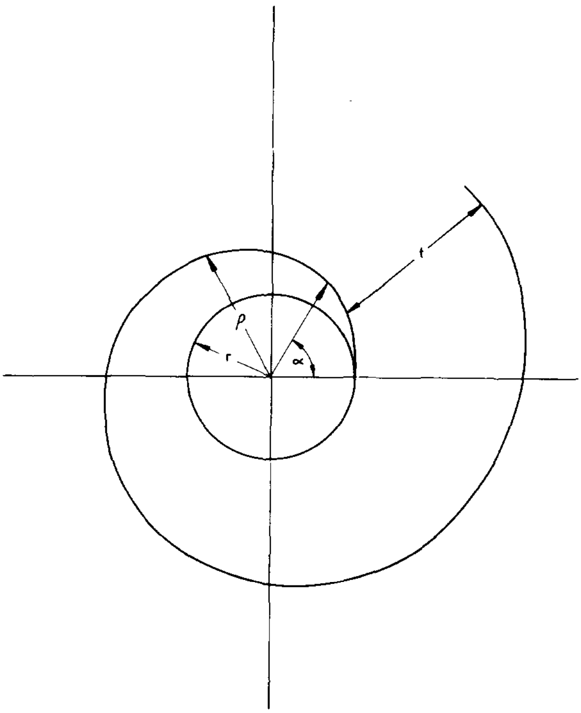


Fig. 1. Geometry of the spiral.

$$L = \pi(n^2t + 2rn) \tag{2}$$

Marincic [1] states that the minimal diameter (D_c) of the circle containing the spiral is given by:

$$D_c = \rho_n + \rho_{(n-0.5)} \tag{3}$$

However, it is shown in Fig. 2 that this is not strictly correct. The diameter given by eqn. (3) is shown as AB. The true minimum diameter (D_t) is shown as AC. Let the number of turns to C and A be n' and n respectively. Angles $\angle$OAC and $\angle$ACO are shown in Fig. 2 as ϵ and β respectively. It is shown in the Appendix that:

$$\tan\beta = t/2\pi(r + n't) \tag{4}$$

Using eqn. (1):

$$\mathrm{OP} = (r + n't)\sin\beta = (r + nt)\sin\epsilon \tag{5}$$

Also

$$n - n' = \{\pi - (\epsilon + \beta)\}/2\pi \tag{6}$$

Rearranging eqn. (4):

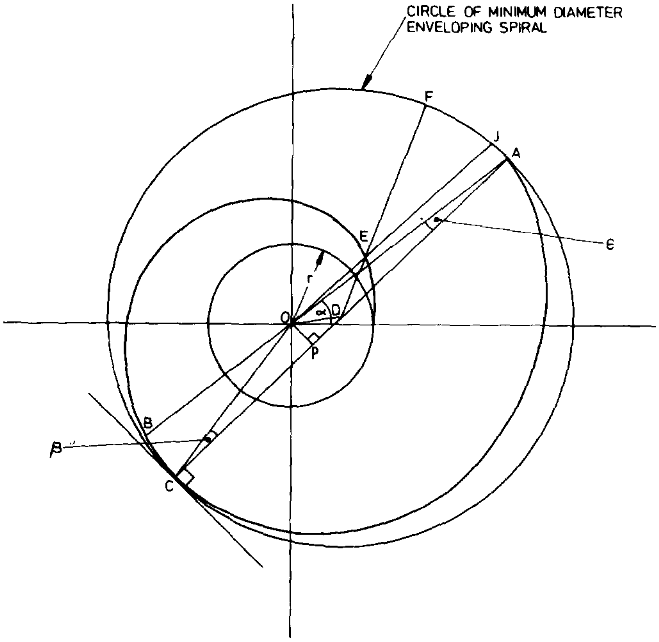


Fig. 2. Envelopment of a spiral by a circle. E is a point on the spiral. Straight lines OE and DE intersect the enveloping circle at points J and F respectively.

$$n' = \frac{1}{2\pi \tan\beta} - \frac{r}{t} \tag{7}$$

From eqn. (5):

$$\epsilon = \sin^{-1} \frac{(r + n't)\sin\beta}{(r + nt)} \tag{8}$$

From eqn. (6):

$$\frac{n - 1/2 + (\epsilon + \beta)/2\pi}{n'} - 1 = 0 \tag{9}$$

Equations (7) - (9) were solved by an iterative method using a Texas SR-52 programmable calculator. For particular values of n, r and t a value was assumed for β and n' calculated using eqn. (7). Then ϵ was evaluated using eqn. (8) and hence the l.h.s. of eqn. (9) was computed. β was adjusted until the l.h.s. of eqn. (9) was very small. The true value for the minimum diameter of the enveloping circle (D_t) was then calculated according to eqn. (10) and also the fractional error ($D_t/D_c - 1$) made in using eqn. (3):

$$D_t = \mathrm{AP} + \mathrm{PC} = (r + n't)\cos\beta + (r + nt)\cos\epsilon \tag{10}$$

Values obtained when $\{n - 1/2 + (\epsilon + \beta)/2\pi\}n' - 1 < 10^{-7}$ are presented in Table 1. The maximum error resulting from the use of eqn. (3)

TABLE 1

Computations of the error resulting from the use of eqn. (3)

n		t/r				
		0.1	0.3	1	3	10
1	n'	0.505	0.512	0.529	0.547	0.560
	D_t/r	2.150	2.452	3.515	6.572	17.31
	ϵ°	0.83	2.10	4.54	6.75	8.09
	β°	0.87	2.37	5.94	10.25	13.56
	$(D_t/D_c - 1)$	1.1×10^{-4}	7.6×10^{-4}	4.2×10^{-3}	1.1×10^{-2}	1.8×10^{-2}
3	n'	2.504	2.508	2.514	2.516	2.518
	D_t/r	2.550	3.651	7.507	18.525	57.09
	ϵ°	0.70	1.44	2.28	2.73	2.94
	β°	0.73	1.56	2.59	3.20	3.48
	$(D_t/D_c - 1)$	7.8×10^{-5}	3.4×10^{-4}	9.0×10^{-4}	1.3×10^{-3}	1.6×10^{-3}
10	n'	9.503	9.504	9.505	9.505	9.505
	D_t/r	3.950	7.851	21.502	60.51	197.0
	ϵ°	0.46	0.68	0.83	0.88	0.90
	β°	0.47	0.71	0.87	0.93	0.95
	$(D_t/D_c - 1)$	3.2×10^{-5}	7.4×10^{-5}	1.1×10^{-4}	1.2×10^{-4}	1.3×10^{-4}

is 1.8% for $n = 1$ and $t/r = 10$ which is an extreme case. In the nickel–cadmium cell typical values are $n = 7$ and $t/r = 0.5$ for which the calculated error in using eqn. (3) is <0.02%. Equation (3) is therefore an extremely close approximation and will be used subsequently for the calculation of n.

It can be seen in Table 1 that ϵ and β are usually small and hence the approximation $\beta = \sin\beta = \tan\beta$, $\epsilon = \sin\epsilon$ for angles in radians may be made to allow eqns. (4) - (6) to be solved. Equations (4) and (5) then become:

$$\beta = t/2\pi(r + n't) \tag{11}$$

$$(r + n't)\beta = (r + nt)\epsilon \tag{12}$$

Equating $(r + n't)\beta$ in eqns. (11) and (12):

$$\epsilon = t/2\pi(r + nt) \tag{13}$$

Substituting for ϵ and β from eqns. (11) and (13) into eqn. (6) and writing $2\pi(n - 1/2) + t/2\pi(r + nt) = K$ leads to:

$$4\pi^2 t(n')^2 + (4\pi^2 r - 2\pi Kt)n' - (t + 2\pi Kr) = 0 \tag{14}$$

This is a quadratic equation in n'. Hence given n, r and t, n' may be calculated from eqn. (14) and β and ϵ may be determined from eqns. (11) and (13).

3. Distances between the spiral and the outer circle

3.1. Theory

The required distances EF and EJ for the determination of possible electrode overlap are shown in Fig. 2. The centre of the enveloping circle is D. The essential parts of Fig. 2 required for a geometric analysis are shown in Fig. 3. It should be noted that OC = $\rho_{n'}$, OA = ρ_n and JD = CD = $D_t/2$.

The angles λ, β, ϵ, α, θ and γ are shown in Fig. 3 and all other angles may be expressed in terms of these six. It can be seen using eqn. (1) that:

$$\mathrm{OE} = r + \{(n-1) + (\lambda - \alpha)/2\pi\}t \tag{15}$$

Furthermore

$$\mathrm{OG} = \mathrm{OD}\cos(\lambda - \theta) \tag{16}$$

$$\mathrm{GE} = \mathrm{OD}\sin(\lambda - \theta)\cot(-\lambda + \alpha + \epsilon + \gamma) \tag{17}$$

But OE = OG + GE. Therefore from eqns. (15) - (17):

$$r + \{(n-1) + (\lambda - \alpha)/2\pi\}t = \mathrm{OD}\,\{\cos(\lambda - \theta) + \sin(\lambda - \theta)\cot(-\lambda + \alpha + \epsilon + \gamma)\} \tag{18}$$

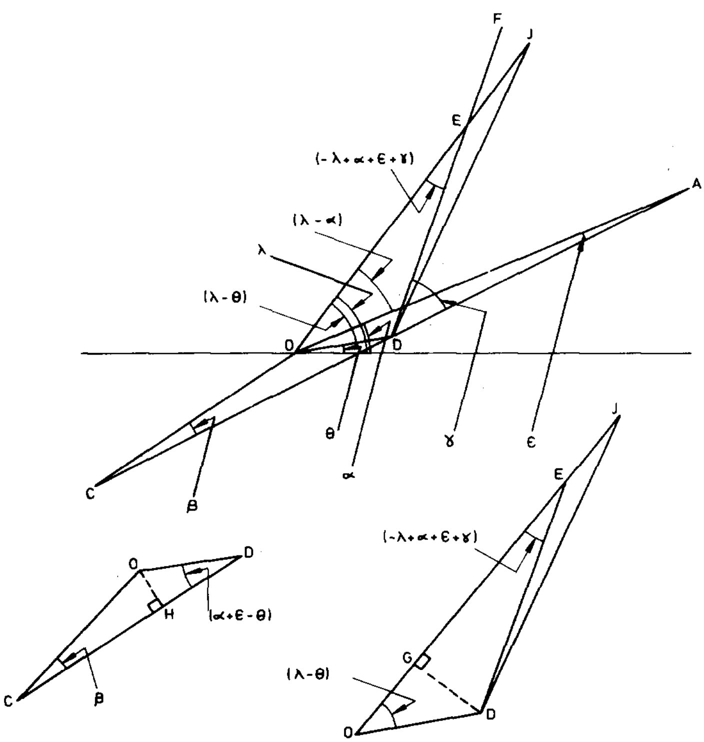


Fig. 3. The essential parts of Fig. 2 for a mathematical analysis.

$$\text{OH} = \rho_{n'} \sin\beta = \text{OD} \sin(\alpha + \epsilon - \theta) \tag{19}$$

$$\text{OD} = (\rho_{n'} \sin\beta)/\sin(\alpha + \epsilon - \theta) \tag{20}$$

But CH + HD = $D_t/2$ and therefore

$$\rho_{n'} \cos\beta + (\rho_{n'} \sin\beta) \cot(\alpha + \epsilon - \theta) = D_t/2 \tag{21}$$

$$\theta = \alpha + \epsilon - \tan^{-1} \frac{\rho_{n'} \sin\beta}{D_t/2 - \rho_{n'} \cos\beta} \tag{22}$$

Also

$$\text{ED} = \text{OD} \sin(\lambda - \theta)/\sin(-\lambda + \alpha + \epsilon + \gamma) \tag{23}$$

Next an expression for OJ will be derived:

$$\angle\text{OJD} = \sin^{-1} \frac{\text{OD} \sin(\lambda - \theta)}{\text{JD}} \tag{24}$$

But OJ = OG + GJ and therefore:

$$\text{OJ} = \text{OD} \cos(\lambda - \theta) + \frac{D_t}{2} \cos\left\{\sin^{-1} \frac{\text{OD} \sin(\lambda - \theta)}{D_t/2}\right\} \tag{25}$$

The above equations then allow the determination of the required quantities EF and EJ for given values of D_c, r, t and γ as follows: (i) calculate n using eqns. (1) and (3); (ii) calculate n' by solving eqn. (14); (iii) calculate β and ϵ using eqns. (11) and (13); (iv) calculate D_t using eqn. (10); (v) calculate α from the decimal part of n; (vi) calculate θ using eqns. (22) and (1); (vii) calculate OD using eqns. (20) and (1); (viii) use an iterative method for solving eqn. (18) *i.e.* insert a λ value and calculate the l.h.s. and r.h.s. of eqn. (18). Adjust λ until (l.h.s. — r.h.s.) is very small; (ix) calculate EF = $D_t/2$ — ED using eqn. (23); (x) calculate EJ = OJ — OE using eqns. (25) and (15). Steps (i) - (x) can be performed on a programmable calculator. In all the calculations which were done the more positive root of eqn. (14) was the correct one and hence the other root may be ignored.

3.2. Results

Calculated values of $\lambda - \alpha$, EF/t and EJ/t are shown in Table 2 for values of D_t/t from 2.34 - 43.8 and γ from 0 - 180°.

A plot of EJ/t *vs.* $\lambda - \alpha$ is shown in Fig. 4. Little dependence on D_t/t is indicated. A linear regression analysis was performed on the data $0° \leqslant \lambda - \alpha < 121°$ resulting in the equation (for angles in degrees):

$$\text{EJ}/t = 1.0027 - 0.007078(\lambda - \alpha) \tag{26}$$

The coefficient of determination was 0.998 indicating that the points correlate well with the straight line in the range covered. The line given by eqn. (26) is shown broken in Fig. 4.

TABLE 2

The distances EF and EJ between the spiral and the outer circle (angles are in degrees)

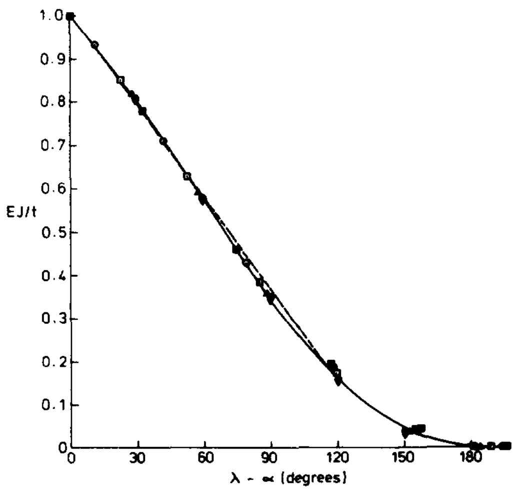
D_t/t	γ	$\lambda-\alpha$	EF/t	EJ/t	D_t/t	γ	$\lambda-\alpha$	EF/t	EJ/t
2.3437	0	–	1.08398	–	7.8879	0	–	1.00285	–
	30	–	0.97471	–		–	0	–	1
	–	0	–	1		30	27.62	0.81839	0.82044
	60	32.78	0.73210	0.78027		60	57.20	0.59106	0.59296
	90	74.63	0.43990	0.46023		90	88.11	0.35803	0.35885
	120	117.09	0.19358	0.19576		120	120.04	0.16257	0.16268
	150	157.96	0.04434	0.04435		150	152.43	0.03919	0.03919
	180	196.01	0	0		180	184.64	0	0
2.6726	0	–	1.04640	–	20.057	0	–	1.00040	–
	–	0	–	1		–	0	–	1
	30	11.12	0.90607	0.93321		30	29.29	0.80834	0.80863
	60	41.91	0.68133	0.71081		60	59.10	0.57759	0.57785
	90	78.81	0.41684	0.43966		90	89.40	0.34700	0.34712
	120	118.04	0.18656	0.18803		120	120.08	0.15706	0.15708
	150	156.99	0.04340	0.04341		150	150.97	0.03800	0.03799
	180	193.96	0	0		180	181.82	0	0
4.0128	0	–	1.01387	–	43.751	0	–	1.00008	–
	–	0	–	1		–	0	–	1
	30	22.78	0.84505	0.85456		30	29.70	0.80562	0.80568
	60	52.24	0.62216	0.63136		60	59.62	0.57361	0.57367
	90	84.83	0.38098	0.38500		90	89.75	0.34358	0.34361
	120	119.46	0.17298	0.17349		120	120.05	0.15528	0.15529
	150	154.69	0.04122	0.04122		150	150.45	0.03759	0.03759
	180	189.19	0	0		180	180.83	0	0

Plots of EF/t *vs.* D_t/t are shown in Fig. 5 for values of γ from 0 - 150°. A considerable dependence on D_t/t is shown in this case. In order to determine EF/t at low D_t/t values it was found convenient to plot EF/t *vs.* $(D_t/t - 2)^{-1}$ as shown in Fig. 6. This resulted in smooth curves from which intermediate points could be ascertained more precisely than from plots of D_t/t *vs.* EF/t (Fig. 5). Using the family of curves of the type in Fig. 6 for γ = 0 - 150° the curves shown in Fig. 7 were plotted of EF/t *vs.* γ for D_t/t values of 2.35, 3.00 and 40.0. Each curve approximates to a straight line in the range $30 \leqslant \gamma \leqslant 120$. Linear regression analyses were performed on the data in Table 2 over this range. The following equation may be written:

$$\mathrm{EF}/t = A\gamma + B \quad (30 \leqslant \gamma \leqslant 120) \tag{27}$$

A and B values are listed in Table 3. These values are plotted *vs.* $(D_t/t - 2)^{-1}$ in Fig. 8. As can be seen the points for A *vs.* $(D_t/t - 2)^{-1}$ lie close to a straight line. A linear regression analysis revealed the relationship:

$$A = -0.000530(D_t/t - 2)^{-1} - 0.00726 \tag{28}$$



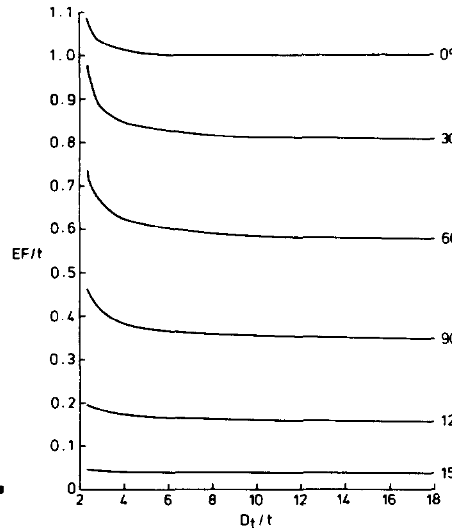
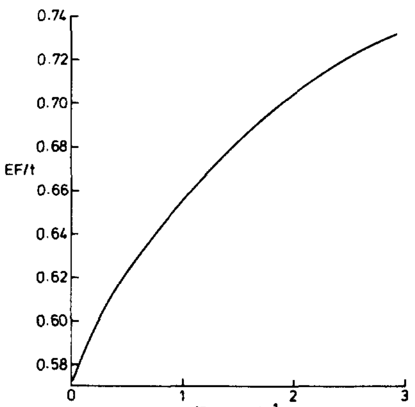


Fig. 4. EJ/t *vs.* $\lambda - \alpha$. D_t/t values: ▪, 2.34; ○, 2.67; □, 4.01; △, 7.89; •, 20.1; ▽, 43.8. Broken line represents the best straight line through the points $0° \leqslant \lambda - \alpha < 121°$.

Fig. 5. EF/t *vs.* D_t/t at the various values of γ indicated on the curves.



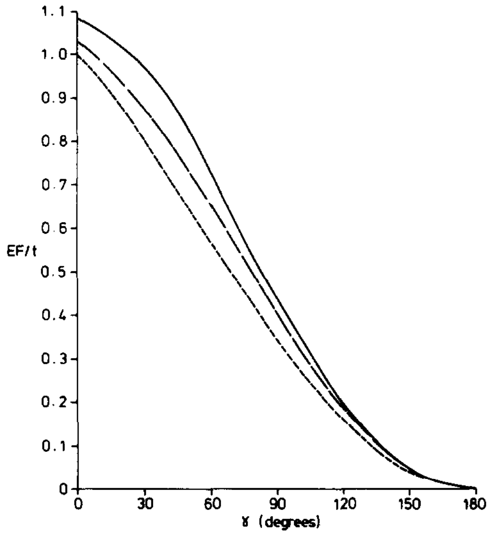


Fig. 6. EF/t *vs.* $(D_t/t - 2)^{-1}$ for $\gamma = 60°$.

Fig. 7. EF/t *vs.* γ for the following D_t/t values: ——, 2.35; — — —, 3.00; -------, 40.0.

Hence for D_t/t values in the range 2.34 - 43.8, A may be determined from eqn. (28) and B by graphical interpolation from Fig. 8 for substitution into eqn. (27).

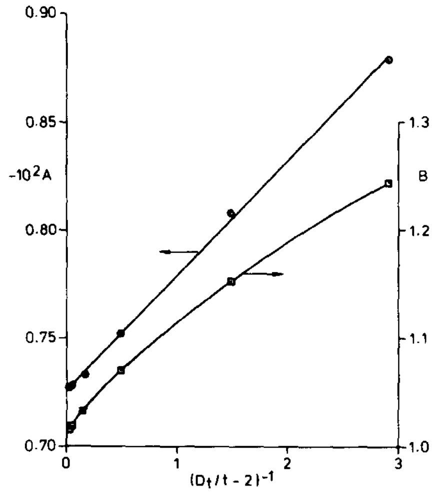


Fig. 8. A and B *vs.* $(D_t/t - 2)^{-1}$.

TABLE 3

Coefficients of eqn. (27)

D_t/t	A (degree^{-1})	B
2.3437	−0.00879	1.244
2.6726	−0.00808	1.153
4.0128	−0.00752	1.070
7.8879	−0.00733	1.033
20.057	−0.00728	1.019
43.751	−0.00727	1.015

4. The spiral-wound construction

The mandrel on which the pack is wound is usually split to allow the use of a single piece of separator (Fig. 9). The length of separator in the split of the mandrel has not been taken into account in the subsequent calculations. Although a single piece of separator is generally used, it is treated here as two pieces, 1 and 2, with compressed thicknesses δ_1 and δ_2 respectively making the analysis more general.

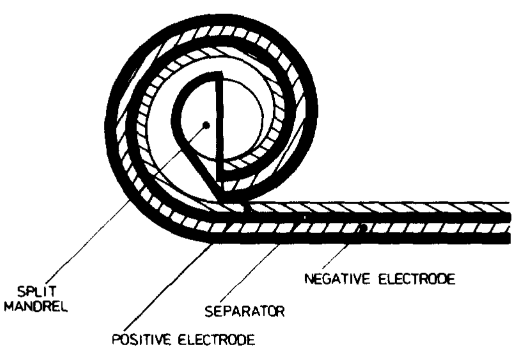


Fig. 9. Winding the components onto a split mandrel.

5. Cases with no electrode overlap at the inside or outside of the coil pack

There are four basic cases corresponding to the positive or negative electrode starting at the centre and the positive or negative ending at the outside

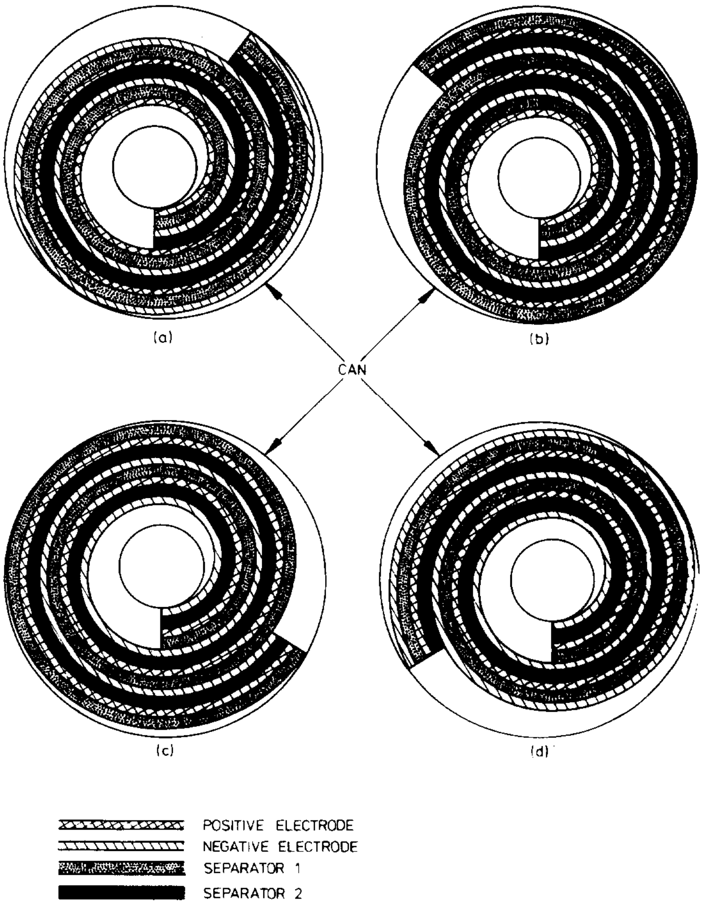


Fig. 10. (a) Case 1: positive starts at centre and negative ends outside. (b) Case 2: positive starts at centre and ends outside. (c) Case 3: negative starts at centre and positive ends outside. (d) Case 4: negative starts at centre and ends outside.

of the pack. The can is the negative terminal and hence if the positive finishes at the outside it must be insulated from the can by a layer of separator. The four cases are shown in Fig. 10.

Let the mandrel have a radius r_c and the positive, separator 1, negative and separator 2 have thicknesses δ_p, δ_1, δ_N and δ_2 and lengths L_p, L_1, L_N and L_2 respectively. In this and subsequent calculations it will be assumed that the length of a flat piece of electrode or separator is equal to the length of a plane through the centre of the component when it is bent around the mandrel, *i.e.* it compresses at the inside and stretches at the outside. Using eqn. (2) for the length of a spiral the following relationships follow:

$$L_p = \pi[(n-a)^2 t + 2\{r_c + \delta_p/2 + d(\delta_N + \delta_2)\}(n-a)] \tag{29}$$

$$L_1 = \pi[(n-a)^2 t + 2\{r_c + \delta_p + \delta_1/2 + d(\delta_N + \delta_2)\}(n-a)] \tag{30}$$

TABLE 4

Values of the coefficients in eqns. (29) - (32), (34), (51), (53) - (55), (58) and (63)

Constant	Case			
	1	2	3	4
a	0	0	0	1
d	0	0	1	1
h	0	1	0	0
k	1	1	0	1

$$L_N = \pi[(n-h)^2 t + 2\{r_c + (1-d)(\delta_p + \delta_1) + \delta_N/2\}(n-h)] \quad (31)$$

$$L_2 = \pi[(n-k)^2 t + 2\{r_c + (1-d)(\delta_p + \delta_1) + \delta_N + \delta_2/2\}(n-k)] \quad (32)$$

Values for the coefficients a, d, h and k are given in Table 4. Also

$$t = \delta_p + \delta_1 + \delta_N + \delta_2 \quad (33)$$

The number of turns n of the component at the outside of the coil pack can be determined by application of eqns. (1) and (3):

$$n = \frac{2D_c - 4\{r_c + (1-a)(\delta_p + \delta_1) + (1-h)\delta_N + (1-k)\delta_2\} + t}{4t} \quad (34)$$

6. Cases with electrode overlap at the centre of the coil pack

It is apparent in Fig. 10 that there is void space between the mandrel and the coil in which further positive or negative electrode could be incorporated.

6.1. Cases 1 and 2

The two possibilities for filling the void space are shown in Fig. 11 depending upon whether the positive or negative electrode protrudes into the void space.

Positive filling void space

The distance between the mandrel and the positive electrode d_1 can be determined using eqn. (1):

$$d_1 = \sigma t/2\pi \quad (35)$$

σ is the angle shown in Fig. 11.

Let the positive protrude into the void space making an angle ψ with the negative. Then the maximum protrusion ψ_m is given by (Fig. 11):

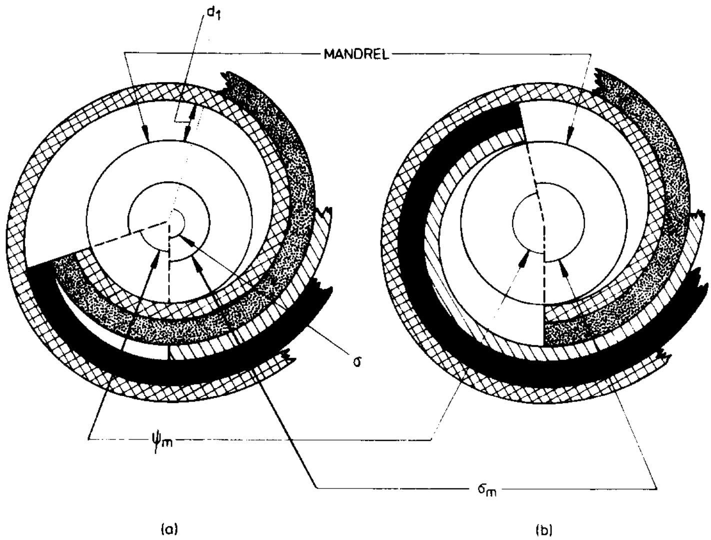


Fig. 11. Filling of void at centre of pack (a) by positive and (b) by negative. Key as for Fig. 10.

$$\psi_m = (2\pi - \sigma_m) \tag{36}$$

where (using eqn. 35):

$$\delta_2 + \delta_p + \delta_1 = \sigma_m t/2\pi \tag{37}$$

Eliminating σ_m from eqns. (36) and (37):

$$\psi_m = 2\pi\{1 - (\delta_2 + \delta_p + \delta_1)/t\} \tag{38}$$

This is equivalent to a length of positive L'_p where

$$L'_p = (r_c + \delta_p/2)\psi_m$$

$$\text{i.e. } L'_p = 2\pi(r_c + \delta_p/2)\{1 - (\delta_2 + \delta_p + \delta_1)/t\} \tag{39}$$

Similarly the corresponding length of separator 1 is

$$(L'_1)_p = 2\pi(r_c + \delta_p + \delta_1/2)\{1 - (\delta_2 + \delta_p + \delta_1)/t\} \tag{40}$$

Separator 2 protrudes into the void space but continues to follow a spiral with ψ_m corresponding to the n value n_m (n_m has a negative value).

$$n_m = -\psi_m/2\pi = (\delta_2 + \delta_p + \delta_1)/t - 1 \tag{41}$$

This corresponds to a length of separator 2 given by:

$$(L'_2)_p = -\pi[\{(\delta_2 + \delta_p + \delta_1)/t - 1\}^2 t + 2(r_c + \delta_p + \delta_1 + \delta_N + \delta_2/2)\{(\delta_2 + \delta_p + \delta_1)/t - 1\}] \tag{42}$$

Negative filling the void space

Let the negative protrude into the void space with a maximum length L'_N. The equation equivalent to eqn. (38) is now:

$$\psi_m = 2\pi\{1-(\delta_N + \delta_2)/t\} \tag{43}$$

This leads to (*cf.* eqn. 42):

$$L'_N = -\pi[\{(\delta_N + \delta_2)/t - 1\}^2 t + 2(r_c + \delta_p + \delta_1 + \delta_N/2)\{(\delta_N + \delta_2)/t - 1\}] \tag{44}$$

Similarly the corresponding length of separator 2 is:

$$(L'_2)_N = -\pi[\{(\delta_N + \delta_2)/t - 1\}^2 t + 2(r_c + \delta_p + \delta_1 + \delta_N + \delta_2/2)\{(\delta_N + \delta_2)/t - 1)\}] \tag{45}$$

6.2. Cases 3 and 4

The treatment is as in the above section except that the positive and separator 1 are interchanged with the negative and separator 2 respectively.

Positive filling void space

The relevant previous equations are (44) and (45):

$$L'_p = -\pi[\{(\delta_p + \delta_1)/t - 1\}^2 t + 2(r_c + \delta_N + \delta_2 + \delta_p/2)\{(\delta_p + \delta_1)/t - 1)\}] \tag{46}$$

$$(L'_1)_p = -\pi[\{(\delta_p + \delta_1)/t - 1\}^2 t + 2(r_c + \delta_N + \delta_2 + \delta_p + \delta_1/2)\{(\delta_p + \delta_1)/t - 1)\}] \tag{47}$$

Negative filling the void space

The relevant previous equations are (39), (40) and (42):

$$L'_N = 2\pi(r_c + \delta_N/2)\{1-(\delta_1 + \delta_N + \delta_2)/t\} \tag{48}$$

$$(L'_2)_N = 2\pi(r_c + \delta_N + \delta_2/2)\{1-(\delta_1 + \delta_N + \delta_2)/t\} \tag{49}$$

$$(L'_1)_N = -\pi[\{(\delta_1 + \delta_N + \delta_2)/t - 1\}^2 t + 2(r_c + \delta_N + \delta_2 + \delta_p + \delta_1/2)\{(\delta_1 + \delta_N + \delta_2)/t - 1\}] \tag{50}$$

7. Cases with electrode overlap at the outside of the coil pack

Referring to Fig. 10 it can be seen that there is void space between the coil and the can in which further positive or negative electrode could be incorporated.

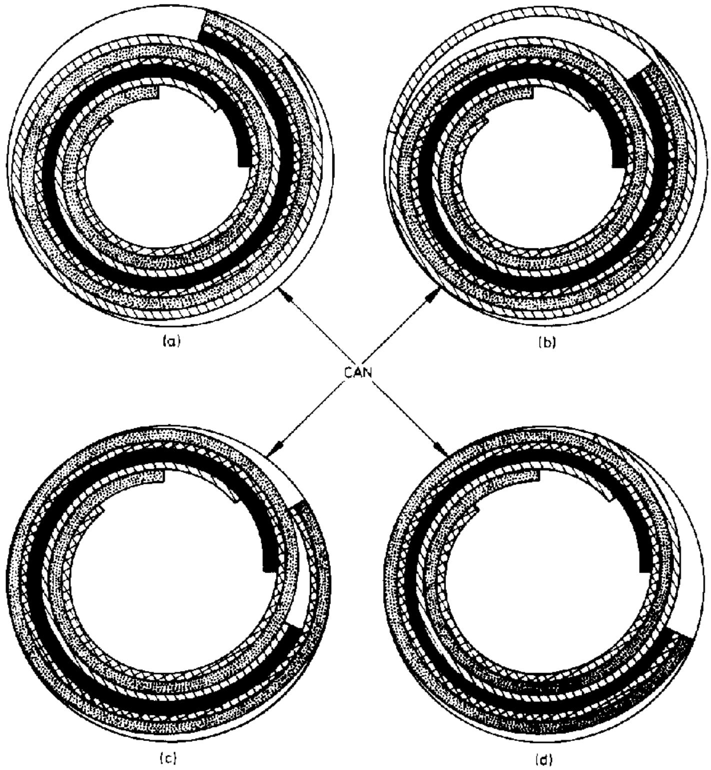


Fig. 12. Cases 1 and 4. Filling of void between pack and can (a) by positive and (b) by negative. Cases 2 and 3. Filling of void between pack and can (c) by positive and (d) by negative. Key as for Fig. 10.

7.1. Cases 1 and 4

The two possibilities for filling the outer void space are shown in Figs. 12(a) and 12(b).

In the analysis presented earlier (eqns. 15 - 25) r may be expressed in terms of r_c, δ_p, δ_1, δ_N and δ_2 for the negative electrode:

$$r = r_c + (1 - a)(\delta_p + \delta_1) + \delta_N/2 \tag{51}$$

Positive filling void space

The gap between the pack and the can which may be filled by separators 1 and 2 plus the positive electrode is equivalent to EJ in Fig. 2. The maximum penetration of the positive into the void space $(\lambda_m - \alpha)$ results when:

$$\mathrm{EJ} = \delta_1 + \delta_p + \delta_2 \quad (\lambda = \lambda_m) \tag{52}$$

$\lambda_m - \alpha$ may be determined by solution of eqns. (52) and (26). The maximum length of overlap by the positive (L_p'') can be calculated using eqn. (2):

$$L_p'' = (\lambda_m - \alpha)\{r_c + \delta_p/2 - a(\delta_1 + \delta_p) + nt + (\lambda_m - \alpha)t/4\pi\} \tag{53}$$

Similarly the corresponding length of separator 1 is L_1'' where:

$$L_1'' = (\lambda_m - \alpha)\{r_c + \delta_p + \delta_1/2 - a(\delta_1 + \delta_p) + nt + (\lambda_m - \alpha)t/4\pi\} \tag{54}$$

And the corresponding length of separator 2 is L_2'' where:

$$L_2'' = (\lambda_m - \alpha)\{r_c - \delta_2/2 - a(\delta_p + \delta_1) + nt + (\lambda_m - \alpha)t/4\pi\} \tag{55}$$

Negative filling void space

The relevant gap between the pack and the can which may be filled with negative electrode is EF in Fig. 2. The maximum penetration of the negative into the void space (γ_m) is given by:

$$\mathrm{EF} = \delta_N \qquad (\gamma = \gamma_m) \tag{56}$$

γ_m may be determined by solution of eqns. (56) and (27). The maximum overlap of the negative L_N'' can be calculated as

$$L_N'' = D_t \gamma_m / 2 \tag{57}$$

Note that D_t here is less than D_c by an amount approximately equal to the negative thickness because D_t is calculated using eqn. (10) where r is given by eqn. (51), *i.e.* calculations are based on a plane through the centre of the negative electrode.

7.2. Cases 2 and 3

The two possibilities for filling the outer void space are shown in Figs. 12(c) and 12(d).

In the earlier analysis (eqns. 15 - 25) r may be expressed as follows:

$$r = r_c + d(\delta_n + \delta_2) + \delta_p + \delta_1/2 \tag{58}$$

Positive filling void space

The gap between the pack and the can which may be filled by separator 1 plus the positive electrode is equivalent to EF in Fig. 2. The maximum penetration of the positive into the void space (γ_m) is given by:

$$\mathrm{EF} = \delta_1 + \delta_p \qquad (\gamma = \gamma_m) \tag{59}$$

γ_m may be determined by solution of eqns. (59) and (27). The maximum overlap of the positive L_p'' can be calculated as:

$$L_p'' = (D_t - \delta_1 - \delta_p)\gamma_m / 2 \tag{60}$$

Similarly the corresponding length of separator 1 is L_1'' where:

$$L_1'' = D_t \gamma_m / 2 \tag{61}$$

Negative filling void space

The relevant gap between the pack and the can which may be filled by the negative electrode is EJ in Fig. 2. The maximum penetration into the void space $(\lambda_m - \alpha)$ is given by:

$$\mathrm{EJ} = \delta_N \qquad (\lambda = \lambda_m) \tag{62}$$

λ_m may be determined by solution of eqns. (62) and (26). The maximum overlap of the negative L''_N can be calculated using eqn. (2) as:

$$L_N = (\lambda_m - \alpha)\{r_c + \delta_N/2 - h(\delta_2 + \delta_N) + nt + (\lambda_m - \alpha)t/4\pi\} \tag{63}$$

8. Calculation of total component lengths

The steps used in the calculations were as follows:

(i) Determine n using eqn. (34) for one of cases 1 - 4.

(ii) Calculate L_p, L_1, L_N and L_2 using eqns. (29) - (32).

(iii) As steps (ii) - (vii) of Section 3.1 using eqns. (51) and (58) for r.

(iv) Calculate the partial component lengths L'_p, $(L'_1)_p$, $(L'_2)_p$, L'_N, $(L_1)_N$ and $(L'_2)_N$ using the equations in Section 6 for overlap at the centre of the coil pack.

(v) Calculate the partial component lengths L''_p, L''_1, L''_2 and L''_N using the equations in Section 7 for overlap at the outside of the coil pack.

(vi) The maximum total component lengths may be determined by adding up the calculated partial lengths as follows:

(a) *Positive filling void spaces at centre and outside of pack*

$$\bar{L}_p = L_p + L'_p + L''_p$$

$$\bar{L}_N = L_N$$

$$\bar{L}_1 = L_1 + (L'_1)_p + L''_1$$

$$\bar{L}_2 = L_2 + (L'_2)_p + L''_2$$

(b) *Positive filling centre void space, negative filling void between pack and can*

$$\bar{L}_p = L_p + L'_p$$

$$\bar{L}_N = L_N + L''_N$$

$$\bar{L}_1 = L_1 + (L'_1)_p$$

$$\bar{L}_2 = L_2 + (L'_2)_p$$

(c) *Negative filling centre void space: positive filling void between pack and can*

$$\bar{L}_p = L_p + L''_p$$

$$\bar{L}_N = L_N + L'_N$$

$$\bar{L}_1 = L_1 + (L'_1)_N + L''_1$$

$$\bar{L}_2 = L_2 + (L'_2)_N + L_2$$

(d) Negative filling void spaces at the centre and outside of the pack

$$\bar{L}_p = L_p$$

$$\bar{L}_N = L_N + L'_N + L''_N$$

$$\bar{L}_1 = L_1 + (L'_1)_N$$

$$\bar{L}_2 = L_2 + (L'_2)_N$$

Conclusion

Equations are presented from which the component lengths may be calculated for the four cases corresponding to the permutations of the two electrodes starting at the inside and ending at the outside. These result in gaps between the mandrel and the coil pack and the latter and the can which may be filled with active material for maximized cell capacity. This leads to four variations on the basic cases, resulting in 16 combinations altogether.

The calculations may be readily performed on a programmable calculator or computer.

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Reference

1 N. Marincic, J. Appl. Electrochem., 6 (1976) 51.

Nomenclature

A, B	constants in eqn. (27),
a, d, h, k	see Table 4,
D_c	specified internal can diameter containing coil pack,
D_t	minimum circle diameter containing spiral,
EF, EJ	see Fig. 2,
K	$= 2\pi(n - 1/2) + t/2\pi(r + nt)$,
L	length of spiral,

L_1, L_2, L_p, L_N	length of separator 1, separator 2, positive, negative, with no overlap at centre or between pack and can,
$L'_p, (L'_1)_p, (L'_2)_p$	length of positive protruding into the centre void space and the associated lengths of separators 1 and 2,
$L'_N, (L'_1)_N, (L'_2)_N$	as above but for negative,
L''_p, L''_1, L''_2	length of positive protruding into the void space between pack and can and the associated lengths of separators 1 and 2,
L''_N	length of negative protruding into the void space between pack and can,
$\bar{L}_1, \bar{L}_2, \bar{L}_p, \bar{L}_N$	maximum total length of separator 1, separator 2, positive, negative,
n	total number of turns on spiral; number of turns of the component at the outside of the coil pack,
n'	number of turns to C (Fig. 2),
r	radius of circle on which spiral originates,
r_c	radius of mandrel on which coil is wound,
t	distance by which spiral moves out during one turn,
$\alpha, \gamma, \theta, \lambda$	see Fig. 3,
β	angle $\angle$ACO (Fig. 2),
$\gamma_m, (\lambda_m - \alpha)$	angles corresponding to the maximum penetration of an electrode into void space between pack and can,
ϵ	angle $\angle$OAC (Fig. 2),
δ_p, δ_N	thickness of positive, negative,
δ_1, δ_2	compressed thickness of separator 1, 2,
ρ_n	distance of spiral from centre after n turns.

Appendix

Derivation of eqn. (4)

The relevant part of Fig. 2 is shown in Fig. 13. A line has been drawn making an angle $\Delta\eta$ with OC intersecting the spiral at R.

$$\angle \mathrm{TCS} = \angle \mathrm{OCP} = \beta \tag{64}$$

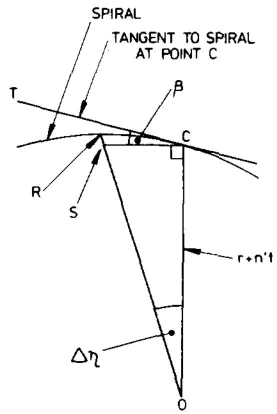
$$\mathrm{RO} = r + \left(n' + \frac{\Delta\eta}{2\pi}\right) t \tag{65}$$

$$\mathrm{OS} = (r + n't) \sec \Delta\eta \tag{66}$$

$$\mathrm{RS} = \mathrm{RO} - \mathrm{OS} = r + \left(n' + \frac{\Delta\eta}{2\pi}\right) t - (r + n't) \sec \Delta\eta \tag{67}$$

$$\mathrm{SC} = (r + n't) \tan \Delta\delta \tag{68}$$

Consider now the situation $\Delta\eta \to 0$ only. Then eqn. (67) becomes:



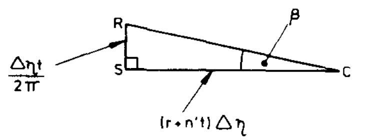


Fig. 13. A section of the spiral required for the derivation of eqn. (4).

Fig. 14. The segment RSC for $\Delta\eta \rightarrow 0$.

$$\mathrm{RS} = \Delta\eta t/2\pi \tag{69}$$

as sec $\Delta\eta = 1 + (\Delta\eta)^2/2$ + higher order terms and eqn. (68) becomes:

$$\mathrm{SC} = (r + n't)\Delta\eta \tag{70}$$

as tan $\Delta\eta = \Delta\eta$ + higher order terms. The segment RSC in Fig. 13 may then be drawn as shown in Fig. 14. Clearly

$$\tan\beta = t/2\pi(r + n't) \tag{4}$$